solution in this way for L' and l' such that there is a solution as in Fig. 1 results in a solution with more than one sheet (points at which x is less than 0 appear on curve HC).

The other cases, where the crater splits up into unconnected areas, are of little interest from the viewpoint of charge interaction and therefore are not considered.

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SPALLING IN STEEL PRODUCED BY EXPLOSION OF A SHEET CHARGE AND COLLISION OF A PLATE

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A specific type of damage (spalling) occurs in strong shock waves and in explosions acting on barriers composed of material of finite tensile strength. The tensile stresses that produce the spalling and fracture arise by interaction between colliding waves. Failure is then always preceded by compression in shock waves.

There are very many papers on such spalling, and one of the early extensive studies [1] gave a detailed description of the phenomena produced in metals by explosive loading. In particular, there was a fairly detailed description of spalling, with an attempt to measure a quantitative characteristic, namely, the critical stress. Subsequent studies [1-13] have provided quantitative criteria and schemes based on finite failure times [2], the dislocation mechanism of failure [12], and a quasiacoustic approximation in terms of stored elastic energy [13].

Shock loading can occur in various ways, e.g., detonation of a block of explosive in contact with an obstacle, collision of a plate, or detonation of a sheet of explosive. Two different situations occur, namely, a planar shock wave propagating into the obstacle or parallel to the free surface, the general case being propagation at some angle α. The critical failure stress may be determined from the difference between the initial speed w_0 of the free surface and the mean speed $\overline{w}$:

$$p_{cr} = (1/2)\rho_0 c_0 \Delta w, \tag{1}$$

where ρ_0 and c_0 are the density and speed of sound in the material. If the shock wave is incident at right angles $\Delta w = w_0 - \overline{w}$ [2], while a shock wave emergent at an angle to the surface gives $\Delta w = w_0 - \overline{w}/\cos\alpha$ [7, 8].

We have examined the effects of sheets of explosives and collision of plates with obstacles made of St.3 steel, with measurement of the thickness and $\overline{w}$ for the fragments. The sheet charge in contact with the obstacle was detonated in such a way that a load traveling with the detonation speed was generated. We used cast Trotyl + Hexogen 50/50. The charges were made as plates 80 × 150 mm² and of thickness 3 and 5 mm. The colliding plates were made of steel of thickness 1.06 and 1.52 mm and moving at speeds of 0.96 and 0.65 km/sec, respectively. These plates were accelerated by sheet charges. The speeds were measured in separate experiments by electrical-contact and optical methods. The obstacles were

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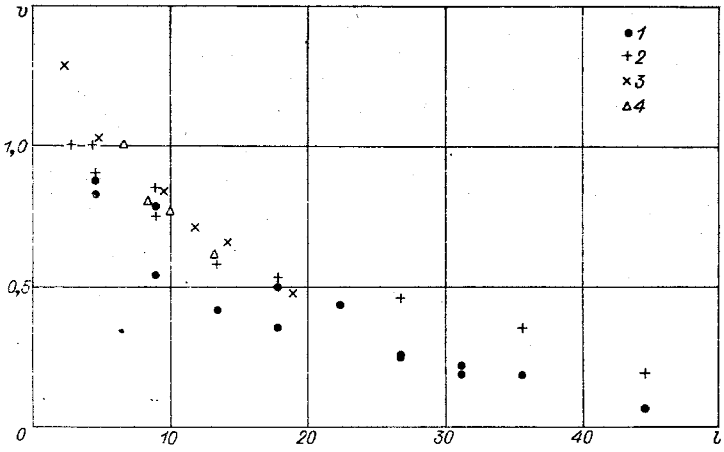

Fig. 1

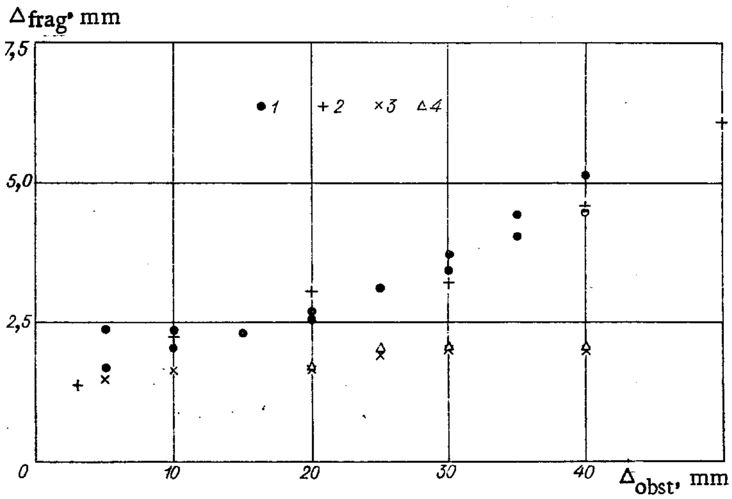

Fig. 2

discs of thicknesses from 3 to 50 mm. Up to a thickness of 20 mm, the diameters were 120 mm, but above 20 mm thickness they were 150 mm. Also, $\overline{w}$ was measured over a baseline of 3 mm by an electrical-contact method. In each case the fragments were trapped for subsequent thickness measurement [7]. The standard deviation in the thickness measurement was 0.1 mm, while that in the fragment speed measurement was 0.015 km/sec. Figures 1 and 2 show the results.

Figure 2 shows the fragment thickness as a function of obstacle thickness, while Fig. 1 shows the speed as a function of thickness in dimensionless terms. The dimensionless velocity v is the ratio of the observed $\overline{w}$ to the mass speed in the contact plane, while the dimensionless thickness l is the ratio of the path traveled by the shock wave in the obstacle to the thickness of the charge sheet exploded in contact with the obstacle or twice the thickness of the incident plate in the collision case. Number 1 in Figs. 1 and 2 corresponds to use of sheet charges of thickness 3 mm, while 2 corresponds to 5-mm charges, 3 corresponds to plates of thickness 1.06 mm, and 4 to plates of thickness 1.52 mm.

The critical stresses were derived from (1); the velocities required here were determined as follows. The value for w_0 for the sheet explosives was taken from [8] by reference to the observed relationship between w_0 and the thickness of the steel obstacle. The w_0 for incident plates were derived by calculation on the basis of the shock-wave damping due to the energy loss on the far side of the plate; the results for the charges of thickness 3-5 mm were $p_{cr} = 257 \pm 18$ kgf/mm², while collision of a plate having $\Delta = 1.06$ mm at $w = 0.96$ km/sec gave $p_{cr} = 815 \pm 47$ kgf/mm², and $\Delta = 1.52$ mm and $w = 0.65$ km/sec gave $p_{cr} = 750 \pm 46$ kgf/mm².

The failure stress in this case is not a constant characteristic; static tests [14, 15] show that the stress is dependent on the time of action. Dynamic loading, as here, also does

TABLE 1

Loading conditions				Source	P_{cr}, kgf/mm², steel	copper	aluminum	$\dot{\varepsilon}$, sec⁻¹
Block of explosive in contact with obstacle								
Charge diameter, mm	Charge height, mm	Obstacle thickness, mm						
Dimensions not stated				[1,11]	112	287	98	$\sim 10^5$
120	40	20—40		[2]	215	365	110	
20	25—36	30—60		[4]	245		145	
Sheet charge in contact with obstacle								
	0,5—5	3—15		[7]			160±12	$\sim 10^6$
	3—5	8—10		[8]			112±19	
	3—5	5		[8]		378±9		
	3—5	5—20		[8]	245±15			
	3—5	3—50		Our results	257±18			
Plate collision								
Material	Plate: Thickness, mm	Plate: Collision speed, km/sec	Thickness, mm					
Copper	5	2,4	24	[9]		1500		$\sim 10^7$
»	2	2,6	10	[2]		780		
Steel	1,06	0,96	5—40	Our results	815±47			
»	1,52	0,65	5—40		750±46			

not involve a constant critical stress, as has been pointed out previously [3, 12]. A formal scheme has been given [3] as a relationship between the failure time and the maximum negative pressure, and it has also proved possible [12] to relate the failure stress to the rate of change of stress in terms of a dislocation mechanism.

Table 1 compares the results for the various methods of loading, and it shows not only reasonably good agreement between the results from different circumstances, but also demonstrates, especially for steel and copper, that p_{cr} is very much dependent on the loading conditions and on the strain rate $\dot{\varepsilon}$, which itself is determined by the gradient in the mass velocity u during interaction between negative-pressure waves ($\dot{\varepsilon} = du/dx|_t$) [2, 3]. Simple estimates [3] imply that charges of Trotyl–Hexogen 50/50 mixture of height 40-50 mm exploded on specimens of thickness 10-40 mm give $\dot{\varepsilon}$ ~ sec⁻¹. The strain rate rises to about ~10^6 sec⁻¹ if the charge thickness is reduced to 3-5 mm. The rate rises to ~10^7 sec⁻¹ for collision with a thin plate, e.g., a plate of thickness 1.52 mm moving at 0.65 km/sec and colliding with another steel plate of thickness 5 mm gives a strain rate in the interaction zone of $1.94 \cdot 10^7$ sec⁻¹. The examples of steel and copper in Table 1 show that p_{cr} increases by a factor of 2.5-3 on increasing the strain rate by about two orders of magnitude, namely, from ~10^5 to ~10^7 sec⁻¹.

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BUCKLING IN AN ELASTIC ROD UNDER A TIME-VARYING LOAD

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A rod subject to a steady heavy load [1] may be replaced by a system with one degree of freedom provided that the motion is examined over a sufficiently long time interval [2]. The rod has to be approximated by a system with a larger number of degrees of freedom [3] if the strong load is aperiodic.

The following equation describes the buckling in a homogeneous elastic rod subject to an alternating heavy load:

$$EIw_{xxxx} + N(t)w_{xx} + \rho F w_{tt} = f(x),\ 0 \leqslant x \leqslant L,\ t > 0, \tag{1}$$

where w is the normal deflection, x and t are the longitudinal coordinate and time, L is rod length, ρ is density, F and I are the constant cross-sectional area and bending rigidity of the rod, E is Young's modulus, N(t) is the given longitudinal force (Fig. 1), and f(x) is a function determined by the given small perturbations or imperfections.

Here N(t) is a continuous monotonically increasing function of time, which increases from zero and runs successively through the critical values for the static case, $N(t) = m^2N_e$, $N_e = \pi^2EIL^{-2}$ (m = 1, 2, ...).

We assume that the hinge-supported rod is at rest before loading; then the initial and boundary conditions take the form

$$w = w_t = 0\ (t=0,\ 0 \leqslant x \leqslant L),\ w = w_{xx} = 0 \quad (x = 0, L,\ t > 0). \tag{2}$$

The solution to (1) and (2) is sought as

$$w = \sum_{m=1}^{\infty} q_m(t) \sin\frac{m\pi x}{L}. \tag{3}$$

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